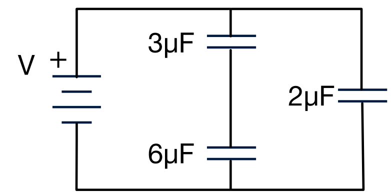
AP2 Circuits

Answer the following questions based on the scheμmatic at right, which shows a 3 μF and 6 μF capacitor connected in series, with a 2 μF capacitor connected in parallel to them. The system of capacitors is connected to a battery of voltage, V.



(a) Rank the potential differences across each capacitor and the battery (1 indicates greatest potential. Give the same rank value for any that have the same potential difference.)

_____$V_{2\mu F}$ _____$V_{3\mu F}$ _____$V_{6\mu F}$ _____$V_{battery}$

(b) Show that the charge on the 3 μF capacitor must be the same value as the charge on the 2 μF capacitor.

(c) Calculate the ratio of the energy stored in the 2 μF capacitor to that of the 3 μF capacitor.

Answers:

(a) 1 $V_{battery}$; 1 $V_{2\mu F}$ because any capacitor in parallel has same potential as source (assuming no other caps in series w/source); 3 $V_{3\mu F}$ same charge as $V_{6\mu F}$ and since V=Q/C, lower C gives higher voltage drop; 4 $V_{6\mu F}$ higher C, lower V for given charge Q.

__1__$V_{2\mu F}$ __3__$V_{3\mu F}$ __4__$V_{6\mu F}$ __1__$V_{battery}$

(b) $Q_2=C_{2\mu F}V_{2\mu F}=(2\mu F)V_{battery}$ (since $C_{2\mu F}$ is in parallel w/battery).
$Q_3=C_{3\mu F}V_{3\mu F}$ where $V_{3\mu F}=(2/3)V_{battery}$ ($V_{3\mu F}+V_{6\mu F}=V_{battery}$ and half the capacitance leads to twice the voltage drop), therefore $Q_{3\mu F}=3\mu F(2/3)V_{battery}=(2\mu F)V_{battery}$

(c) $U=Q^2/2C$, charge is the same for both, so $U_{2\mu F}/U_{3\mu F} = (1/C_{2\mu F})/(1/C_{3\mu F}) = 3/2$

EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

SP: 2.3 The student can estimate numerically quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.E.5.1 The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

AP2 Circuits

A 2 μF, 3 μF, and 6 μF capacitor are connected in series to a 220-volt source. When completely charged, which of the following statements are true? Select two answers.

(A) The charge on the 2 μF capacitor is equal to 440 μC.
(B) The charge on the 3 μF capacitor is equal to 220 μC.
(C) The charge on all three capacitors is the same.
(D) The voltage drop across each of the three capacitors is equal to 220 V.

Answer: (B) and (C)

(A) is false because the voltage across the 2 μF capacitor is not 220 V, so $Q=CV=2\ \mu F \times 220\ V = 440\ \mu C$ is false.
(B) is true since $C_{eq} = 1\ \mu F$, so the total charge is $Q = CV = (1\ \mu F)(220 V) = 220\ \mu C$, and the charge on capacitors in series is the same.
(C) is true because the charge on capacitors in series is the same.
(D) is false because the sum of the voltage drops is 220V; therefore the voltage drop across each capacitor cannot be 220V.

EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors. 5.B.9 Kirchhoff's loop rule describes conservation of energy in electrical circuits. 5.C.3 Kirchhoff's junction rule describes the conservation of electric charge in electrical circuits. Since charge is conserved, current must be conserved at each junction in the circuit.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.E.5.1 The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel. 5.B.9.5 The student is able to use conservation of energy principles (Kirchhoff's loop rule) to describe and make predictions regarding electrical potential difference, charge, and current in steady-state circuits composed of various combinations of resistors and capacitors. 5.C.3.6 The student is able to determine missing values and direction of electric current in branches of a circuit with both resistors and capacitors from values and directions of current in other branches of the circuit through appropriate selection of nodes and application of the junction rule.

AP2 Circuits

Capacitors X, Y, and Z are connected in series to a voltage source and each has a capacitance of C, 2C, and 4C, respectively. When completely charged, which of the following choices gives the correct relationships among the stored energy in the capacitors?

(A) $U_X=2U_Y=4U_Z$
(B) $4U_X=2U_Y=U_Z$
(C) $U_X=U_Y=U_Z$
(D) $U_X=4U_Y=16U_Z$

Answer: (A) $U_X=2U_Y=4U_Z$ because in stored energy in capacitors is given by $U=Q^2/C$. Since the charges have to be the same value in a series configuration, the energy is inversely proportional to the capacitance.

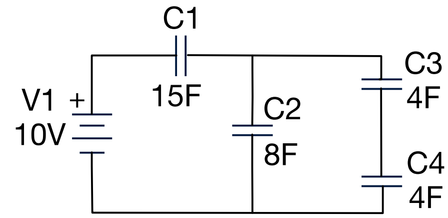
EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.E.5.2 The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

AP2 Circuits

Determine the effective capacitance of the circuit shown.



(A) 19 F
(B) 31 F
(C) 6 F
(D) (184/31) F

Answer: (C) 6 F. The two 4F capacitors in series reduce to 2F; added to the 8F capacitor in parallel gives 10F; 10F & 15F in series gives a total equivalent capacitance of 6F.

EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 4.E.5.1 The student is able to make and justify a quantitative prediction of the effect of a change in values or arrangements of one or two circuit elements on the currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

AP2 Circuits

Capacitor X with a capacitance C is connected to a battery of voltage V. Capacitor Y of capacitance 2C is connected to another battery of voltage 4V. Both capacitors are then disconnected from the batteries and connected to each other in parallel.

(a) What is the overall charge on both capacitors in terms of C and V?
(b) What is the potential difference across the capacitors?
(c) Using your answers from part (b), write an expression for the charge on each capacitor.
(d) How does the energy stored in the capacitors before they were connected in parallel compare to the energy stored after they are connected in parallel? Justify your answer.

Answers:

(a) Charge is conserved when the battery is disconnected, so when in parallel, $Q_{total}=Q_x+Q_y=CV+2C(4V)=9CV$
(b) When disconnected, the charges will redistribute such that the potential across each is the same, but different than their original values. $V_x=V_y=Q_{total}/C_{eq}=(9CV)(C+2C)=3V$ (3 times the original voltage, V)
(c) $Q_x=C_xV_x=C(3V)=3CV$ $Q_y=C_yV_y=2C(3V)=6CV$
(d) $U_{after} < U_{before}$

$$U_{before} = \tfrac{1}{2}CV^2 + \tfrac{1}{2}(2C)(4V)^2 = 16.5CV^2$$

$$U_{after} = \tfrac{1}{2}C_{eq}(V')^2 = \tfrac{1}{2}(3C)(3V)^2 = 13.5CV^2$$

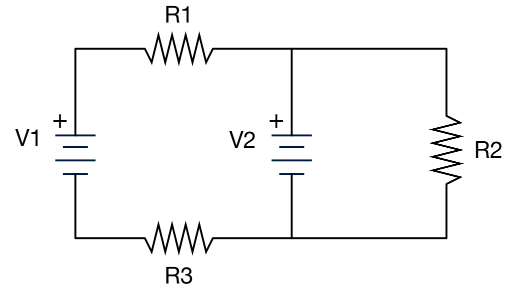
EK: 1.B.1 Electric charge is conserved. The net charge of a system is equal to the sum of the charges of all the objects in the system. 4.E.4 The resistance of a resistor, and the capacitance of a capacitor, can be understood from the basic properties of electric fields and forces, as well as the properties of materials and their geometry.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.1 The student can justify claims with evidence. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 1.B.1.2 The student is able to make predictions, using the conservation of electric charge, about the sign and relative quantity of net charge of objects or systems after various charging processes, including conservation of charge in simple circuits. 4.E.4.1 The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit, based on the geometry of the circuit element and supported by scientific theories and mathematical relationships.

AP2 Circuits

The figure at right shows a circuit with two batteries and three resistors, all labeled. Which of the following actions will increase the current through resistor R_2? (Select two answers.)



(A) Increasing V_1
(B) Increasing V_2
(C) Decreasing R_2
(D) Decreasing R_3

Answers: (B) & (C). The potential across R_2 is completely controlled by V_2. Therefore, to increase the current across R_2, you can either increase V_2 or decrease the resistance of R_2.

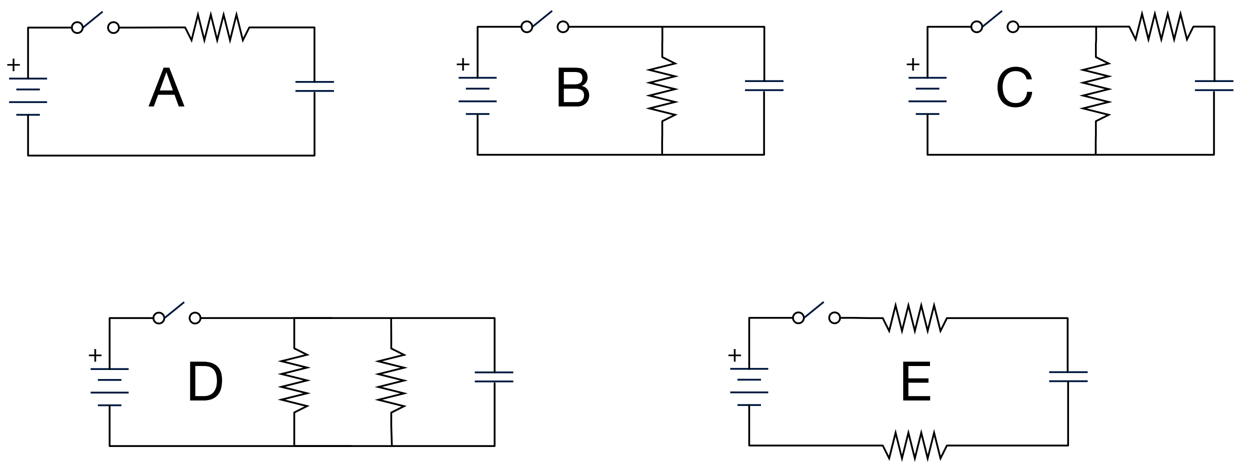
EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena.

LO: 4.E.5.2 The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

AP2 Circuits

The circuits below depict identical batteries, resistors, and capacitors in various configurations. The circuits are initially open, and are all closed at the same time.



A) Rank the current through the battery immediately after the switch is closed from greatest to least.

B) Rank the current through the battery a long time after the switch is closed from greatest to least.

Answers: A) B=D, C, A, E; B) D, B=C, A=E. Note that when the switch is first closed, the capacitor acts like a wire. After a long time, the capacitor acts like an open.

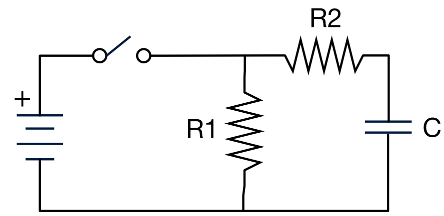
EK: 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

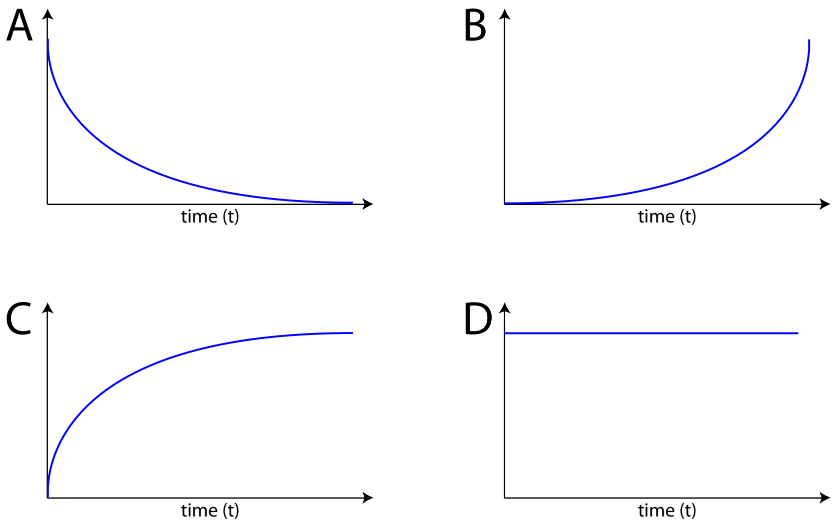
SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 2.2 The student can apply mathematical routines to quantities that describe natural phenomena. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.E.5.2 The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.

AP2 Circuits

The circuit depicted at right shows a battery, two identical resistors, and a capacitor. At time t=0 the switch is closed. The graphs below represent various circuit characteristics as a function of time.





A) Which graph best represents the potential difference across R1?
B) Which graph best represents the current through R1?
C) Which graph best represents the potential difference across R2?
D) Which graph best represents the current through R2?
E) Which graph best represents the potential difference across the capacitor?
F) Which graph best represents the current flow in the capacitor?

Answers:
A) D
B) D
C) A
D) A
E) C
F) A

EK: 4.E.4 The resistance of a resistor, and the capacitance of a capacitor, can be understood from the basic properties of electric fields and forces, as well as the properties of materials and their geometry. 4.E.5 The values of currents and electric potential differences in an electric circuit are determined by the properties and arrangement of the individual circuit elements such as sources of emf, resistors, and capacitors.

SP: 1.4 The student can use representations and models to analyze situations or solve problems qualitatively and quantitatively. 6.4 The student can make claims and predictions about natural phenomena based on scientific theories and models.

LO: 4.E.4.1 The student is able to make predictions about the properties of resistors and/or capacitors when placed in a simple circuit, based on the geometry of the circuit element and supported by scientific theories and mathematical relationships. 4.E.5.2 The student is able to make and justify a qualitative prediction of the effect of a change in values or arrangements of one or two circuit elements on currents and potential differences in a circuit containing a small number of sources of emf, resistors, capacitors, and switches in series and/or parallel.